International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 9
Article Received: *25 July 2023* ***Revised:*** *12 September 2023* ***Accepted:*** *30 September 2023*

Model Predictive Control with Numerical Solution based on Contraction Mapping Method for Stabilization of Vehicle Nonlinear Dynamics

[1] Hiroto Yoshioka, [2] Tomoaki Hashimoto
[1] Osaka Institute of Technology, [2] Osaka Institute of Technology
[1] yoshioka.hiroto@sys-lab.org , [2] tomoaki.hashimoto@oit.ac.jp

Abstract— This paper investigates the nonlinear model predictive control problem for stabilization of unstable vehicle dynamics. Model predictive control (MPC) method is a kind of optimal feedback control method in which the control performance over a finite future is optimized. The contraction mapping algorithm is used for solving the nonlinear model predictive control problem within a short sampling period. A nonlinear tire model is employed to describe the realistic behavior of vehicle motions. The objective of this paper is to propose a nonlinear model predictive control method with a fast numerical solution algorithm called contraction mapping method for designing an automatic vehicle control system. The effectiveness of the proposed method is verified by numerical simulation.

Index Terms— Vehicle control, Nonlinear control, Optimal control, Model predictive control.

I. INTRODUCTION

In recent years, various control problems of vehicle dynamics such as collision avoidance [1], rollover prevention [2], wheel slip control [3], driver assistance control [4] have been studied. In particular, model predictive control (MPC) method is widely used to solve various vehicle control problems. In [5], a linear MPC has been proposed for control of steering and braking in autonomous vehicle navigation. In [6], MPC control method has been proposed to solve the problem with brake torque constraints of electronic mechanical brake mechanism for a vehicle yaw stability. Furthermore, MPC method based on an integrated control algorithm for vehicle in active steering and dynamics yaw control has been proposed in [7].

MPC is known as a well-established control method in which the control input is obtained by solving an open-loop optimal control problem with finite evaluation interval. This optimization procedure is repeated at each sampling instant. Thus, MPC is a type of optimal feedback control in which the control performance over a finite future is optimized and its performance index has a moving initial time and a moving terminal time. MPC is known as one of the most successful control methods because it enables control performance to be optimized while taking constraints on state and control variables into consideration [8]-[12].

Model predictive control problems of vehicle nonlinear dynamics are known to be nonlinear model predictive control (NMPC) problems that need to be solved by numerical algorithm of nonlinear optimal control. Thus, the implementation of NMPC, in which the control problem is solved on-line, poses significant challenges in terms of computational load. This study examines the stabilization problem of unstable vehicle dynamics caused by a collision accident.

In [13], the NMPC problem has been studied for stabilization of vehicle nonlinear dynamics to avoid the second accident after the first collision accident. Then, a fast numerical solution method has been developed in [13] based on C/GMRES algorithm [14] to solve the NMPC problem for stabilization of unstable vehicle dynamics caused by collision accident.

Recently, numerical algorithm called contraction mapping method has been proposed in [15]. It has been shown that this algorithm is faster than the C/GMRES algorithm [14] from the computational point of view. The NMPC problem for stabilization of unstable vehicle dynamics needs to be solved within a short sampling period.

The objective of this paper is to propose a nonlinear model predictive control method with a fast numerical solution algorithm called contraction mapping method for designing an automatic stabilization control system of unstable vehicle dynamics. The effectiveness of the proposed method is verified by numerical simulation.

II. VEHICLE SYSTEM MODEL

In this section, a vehicle system model under the following assumptions is introduced. First, it is assumed that the difference between the vertical loads of the left and right wheels is negligible. Second, it is assumed that rolling and pitching motions are negligible. Finally, it is assumed that the rear tires are not steered. Under those assumptions, the cornering forces of the left and right wheels are equal each other. Thus, a four-wheeled vehicle model can be regarded as a two-wheeled vehicle model. In this study, a two-wheeled vehicle model is considered as shown in Fig. 1, which is equivalent to a four-wheeled vehicle model. The system parameters used in this model are listed in Table 1.

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 9
***Article Received:** 25 July 2023 **Revised:** 12 September 2023 **Accepted:** 30 September 2023*

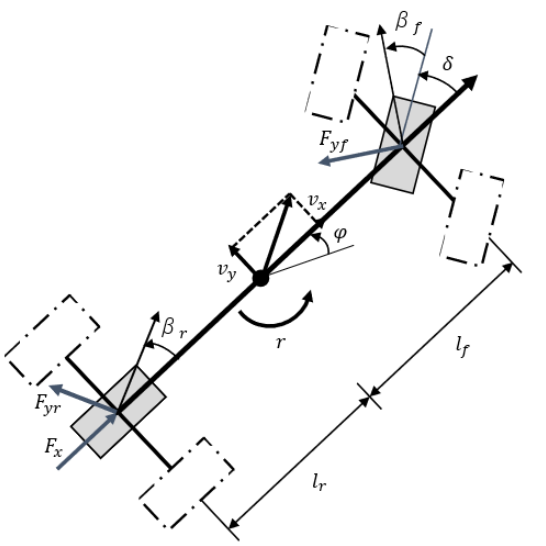

Figure 1: Two-wheeled vehicle model equivalent to four-wheeled vehicle model.

Table 1 System parameters

Notation
m vehicle mass
g gravitational acceleration
μ frictional coefficient
I_{zz} vehicle moment of inertia around z axis
l_f center of mass distance to the front axle
l_r center of mass distance to the rear axle
C_D aerodynamic drag coefficient
A_D effective aerodynamic drag area
F_x driving and braking force
F_{yi} tire lateral force
r vehicle yaw rate
v_x longitudinal velocity in vehicle coordinate
v_y lateral velocity in vehicle coordinate
δ steering angle of front wheel
β_i side slip angle
φ vehicle angle
Y vehicle lateral displacement

The angle between the directions of movement and rotation of the tires is called the slip angle of the tires. The slip angles of the front and rear tires β_f and β_r are given by

$$\beta_f = \tan^{-1}\left(\frac{v_y + l_f r}{v_x}\right) - \delta_f,$$
$$\beta_r = \tan^{-1}\left(\frac{v_y - l_r r}{v_x}\right).$$

In the range where the slip angle is sufficiently small, the lateral force increases in portion to the slip angle. However, the lateral force will saturate and decrease from the maximum value when the slip angle increases beyond a certain value. In other words, the lateral force increases approximately linearly for the first few degrees of slip angle, and then increases non-linearly to a maximum before beginning to decrease. In order to take more realistic tire model into account, we introduce a nonlinear tire model called Magic Formula [16] as follows:

$$F_{yf} = -\frac{1}{2}\mu m g \sin(D \tan^{-1}(\beta_f)),$$
$$F_{yr} = -\frac{1}{2}\mu m g \sin(D \tan^{-1}(\beta_r)).$$

Magic Formula is an empirical formula obtained from experimental data. It is difficult to interpret the formula physically. However, it is more accurate than linear tire model. D is a constant determined to represent the experimental data [16].

For notational simplicity, we introduce the state and input vectors as follows:

$$x(t) = [v_x, v_y, r, Y, \varphi]^{\mathrm{T}},$$
$$u(t) = [F_x, \delta]^{\mathrm{T}}.$$

Using these notations, it is shown in [13] that the system model of vehicle dynamics can be described by the following state equation:

$$\dot{x}(t) = f(x(t), u(t)), \quad (1)$$
$$f(x(t), u(t)) := \begin{bmatrix} \frac{2u_1 - 2F_{yf}\sin(u_2) + m x_2 x_3 - C_D A_D x_1^2}{m} \\ \frac{2F_{yr} + 2F_{yf}\cos(u_2) - m x_1 x_3}{m} \\ \frac{2F_{yf} l_f \cos(u_2) - 2F_{yr} l_r}{I_{zz}} \\ x_1 \sin(x_5) + x_2 \cos(x_5) \\ x_3 \end{bmatrix}.$$

Hereafter, we consider equation (1) as the system model of two-wheeled vehicle motion with nonlinear tire dynamics.

III. NONLINEAR MODEL PREDICTIVE CONTROL

In this section, the nonlinear model predictive control problem of system model (1) is considered. First, the optimal control problem of nonlinear vehicle dynamics is considered. The control input at each time t is determined so as to minimize the following performance index:

$$J = x^{\mathrm{T}}(t+T)Px(t+T) + \int_t^{t+T}\left(x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau)\right)d\tau, \quad (2)$$

where T is the evaluation interval of the performance index, and P, Q, R are weighting coefficients. The optimization problem of (2) subject to equality constraint (1) can be reduced to minimizing the following performance index $\bar{J}$ introduced by using the costate λ associated with the equality constraint.

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 9
***Article Received:** 25 July 2023 **Revised:** 12 September 2023 **Accepted:** 30 September 2023*

$$\bar{J} = x^{\mathrm{T}}(t+T)Px(t+T) + \int_t^{t+T}(x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau) + \lambda^{\mathrm{T}}(\tau)(f(x,u) - \dot{x}))d\tau.$$

Let the Hamiltonian H be defined by

$$H(x(\tau), u(\tau), \lambda(\tau)) = x^{\mathrm{T}}(\tau)Qx(\tau) + u^{\mathrm{T}}(\tau)Ru(\tau) + \lambda^{\mathrm{T}}(\tau)f(x(\tau), u(\tau)).$$

Also, let the terminal cost function φ be defined by

$$\varphi(x(t+T)) = x^{\mathrm{T}}(t+T)Px(t+T).$$

Then, the performance index $\bar{J}$ is rewritten by

$$\bar{J} = \varphi(x(t+T)) + \int_t^{t+T}\left(H(x(\tau), u(\tau), \lambda(\tau)) - \lambda^{\mathrm{T}}(\tau)\dot{x}(\tau)\right)d\tau.$$

Next, we consider the variation in $\bar{J}$ as follows:

$$\delta\bar{J} = \frac{\partial\varphi}{\partial x}\delta x(t+T) + \int_t^{t+T}\left(\left(\frac{\partial H}{\partial x}\right)\delta x + \left(\frac{\partial H}{\partial u}\right)\delta u + \delta\lambda^T\left(\frac{\partial H}{\partial \lambda} - \dot{x}\right) - \lambda^T\delta\dot{x}\right)d\tau$$

It is worth noting that we apply the following integration by parts to the computation on $\delta\bar{J}$.

$$\int_t^{t+T}(\lambda^{\mathrm{T}}(\tau)\,\delta\dot{x}(\tau))d\tau = [\lambda^{\mathrm{T}}(\tau)\,\delta x(\tau)]_t^{t+T} - \int_t^{t+T}\left(\dot{\lambda}^{\mathrm{T}}(\tau)\,\delta x(\tau)\right)d\tau$$

In the above, note that we take $\delta x(t) = 0$ because $x(\tau)$ is fixed at $\tau = t$. Taking the above integration by parts into account, we obtain the variation in $\bar{J}$ as

$$\delta\bar{J} = \left(\frac{\partial\varphi}{\partial x} - \lambda^{\mathrm{T}}(t+T)\right)\delta x(t+T) + \int_t^{t+T}\left(\left(\frac{\partial H}{\partial x} + \dot{\lambda}^{\mathrm{T}}(\tau)\right)\delta x + \left(\frac{\partial H}{\partial u}\right)\delta u + \delta\lambda^T\left(\frac{\partial H}{\partial \lambda} - \dot{x}\right)\right)d\tau$$

On the basis of the variational principle, we obtain the necessary conditions for a stationary value of $\bar{J}$ over the horizon $(t \leq \tau \leq t+T)$ as follows.

$$\dot{x}(\tau) = f(x(\tau), u(\tau)), \qquad (3)$$

$$\dot{\lambda}(\tau) = -\left(\frac{\partial H}{\partial x}\right)^{\mathrm{T}}, \qquad (4)$$

$$\lambda(t+T) = \left(\frac{\partial\varphi}{\partial x}\right)^T, \qquad (5)$$

$$\frac{\partial H}{\partial u} = 0, \qquad (6)$$

Conditions (3)-(6) are called the stationary conditions or Euler-Lagrange equations that must be satisfied for the performance index (2) to be minimized. A well-known difficulty of the nonlinear optimal control is that it results in a nonlinear two-point boundary-value problem that cannot be solved analytically in general. Then, a fast algorithm, called the contraction mapping method, for numerically solving stationary conditions has been proposed in [15]. A brief description of the contraction mapping method applied to this problem is presented in the subsequent discussion.

To solve the stationary conditions in (3)-(6) using numerical algorithm, we must discretize equations (3)-(6) into discrete-time equations. Let the time $\tau \in [t, t+T]$ over the prediction horizon be divided into N steps and k denote the discretized temporal variable. As a result of the discretization approximation, we obtain the discretized stationary conditions over the horizon $(k = 1 \cdots N)$ as follows:

$$x(k+1) = f(x(k), u(k)), \qquad (7)$$
$$\lambda(k) = g(x(k+1), u(k+1), \lambda(k+1)), \qquad (8)$$
$$\lambda(N) = 2Px(N), \qquad (9)$$
$$\frac{\partial H}{\partial u} = 2u^{\mathrm{T}}(k)R + \lambda^{\mathrm{T}}(k)\frac{\partial f}{\partial u} = 0, \qquad (10)$$

Note that $x(1)$ is identical to the present known state $x(t)$. The time-evolutionary equations of x and λ are discretized into forward difference equation in (7) and backward difference equation in (8), respectively. Let the optimization parameters $u(k), (k = 1 \cdots N)$ be unified into a vector denoted by

$$U(t) := [u^{\mathrm{T}}(1) \quad \cdots \quad u^{\mathrm{T}}(N)]^{\mathrm{T}}.$$

In the following, we provide the procedure for computing x, λ, u to satisfy stationary conditions (7)-(10). For the present state $x(1)$ and a given initial solution $U(t)$, $x(k)$ for $(k = 1 \cdots N)$ is calculated recursively from $k = 1$ to $k = N$ by using (7). Next, the terminal costate $\lambda(N)$ is determined from the terminal state $x(N)$ by using (9). Consequently, $\lambda(k)$ for $(k = 1 \cdots N)$ is calculated recursively from $k = N$ to $k = 1$ by using (8). Because x and λ are determined by $x(1)$ and $U(t)$ through equations (7)-(9), equation (10) for $(k = 1 \cdots N)$ can be regarded as a single equation.

$$F(x(1), U(t)) := \begin{bmatrix} 2u^{\mathrm{T}}(1)R + \lambda^{\mathrm{T}}(1)\frac{\partial f}{\partial u} \\ \vdots \\ 2u^{\mathrm{T}}(N)R + \lambda^{\mathrm{T}}(N)\frac{\partial f}{\partial u} \end{bmatrix} = 0 \qquad (11)$$

Because $x(k)$ and $\lambda(k)$ for $(k = 1 \cdots N)$ are uniquely determined through equations (7)-(9) for the given $x(1)$ and $U(t)$, it is worth noting that $x(k)$ and $\lambda(k)$ depend on $x(1)$ and $U(t)$. Hence, it is reasonable to consider the arguments of F as $x(1)$ and $U(t)$.

For a given $x(1)$ and $U(t)$, F is not necessarily equal to zero, so $\|F\|$ is used to evaluate the optimality performance. If $\|F\| = 0$ is satisfied for the given $x(1)$ and $U(t)$, then the stationary conditions are satisfied. Several algorithms have been developed such that $\|F\|$ can be decreased by suitably updating $U(t)$ as discussed below.

A conventional way to update $U(t)$ is to replace $U(t)$ with $U(t) + \alpha s$, known as the steepest descent method, where s is the steepest descent direction and α is the step length satisfying the Armijo condition. For Newton's method, s is given by the Hessian instead of the gradient. However, these methods are computationally expensive, and it was shown that the contraction mapping method is much faster than these methods.

Note that $F(x(1), U(t))$ can be described as

$$F(x(1), U(t)) = AU(t) + b(x(1), U(t)). \qquad (12)$$

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 9
***Article Received:** 25 July 2023 **Revised:** 12 September 2023 **Accepted:** 30 September 2023*

Then, the updating low of $U(t)$ based on the contraction mapping method is given by

$$U(t) = A^{-1}b(x(1), U(t-1)). \qquad (13)$$

More detailed information about the implementation of contraction mapping method is provided in [15].

IV. NUMERICAL SIMULATIONS

In this section, an illustrative example is provided to verify the effectiveness of the proposed method. We consider the situation where a vehicle become unstable motion after the collision accident. Here, we examine the effectiveness of the proposed method to stabilize such an unstable vehicle motion. In this simulation, we set the weighting coefficients and the initial state as follows:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 300 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 500 \end{bmatrix}, \ Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 300 \end{bmatrix}, x(0) = \begin{bmatrix} 10 \\ 2 \\ 0.1 \\ 0 \\ 0 \end{bmatrix},$$

The other parameters employed in the numerical simulations are as follows:
$m = 917$, $g = 9.81$, $\mu = 0.5$, $I_{zz} = 1128$, $l_f = 0.91$, $l_r = 1.64$, $C_D = 0.3$, $A_D = 3$.

In the following, we provide the simulation results to verify the effectiveness of the proposed method. Figures 2-6 show the time responses of state variables using nonlinear model predictive control based on the C/GMRES method and the contraction mapping (CM) method. It is seen that not only lateral velocity and vehicle yaw rate but also lateral displacement and vehicle angle converge to zero and the unstable motion of the vehicle is well stabilized. In other words, not only x_2 and x_3 but also x_4 and x_5 are well stabilized to converge to zero, that means, unstable lateral and yaw motions of vehicle are well stabilized using the proposed method. Furthermore, it is seen that both time responses of state variables are almost similar.

Figures 7-8 show the time responses of control inputs. It is seen that time histories of control inputs using the C/GMRES method and the CM method are similar each other. Figure 9 shows the time response of the optimality errors. It is seen that the optimality error of C/GMRES method is smaller than that of the CM method. Although the optimality in case of CM method is deteriorated than C/GMRES method, the performance of system response is almost same in both cases.

Simulation is performed on a laptop computer (CPU: Intel(R) Core(TM) i7-8550U 1.80 [GHz], Memory: 8.0 [GB], OS: Windows 10, Software: Matlab). The computational times per update (one control cycle) using the C/GMRES method and the CM method are listed in Table 2. It is seen that the computational time of the CM method is smaller than that of the C/GMRES method.

Table 2

Method	Maximum time [ms]	Average time [ms]
C/GMRES	9.850	0.705
CM	0.354	0.416

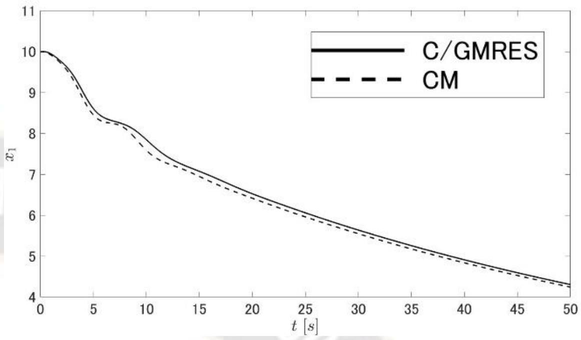

Figure 2: Time responses of x_1.

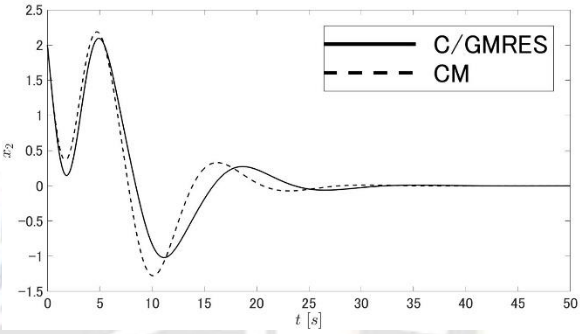

Figure 3: Time responses of x_2.

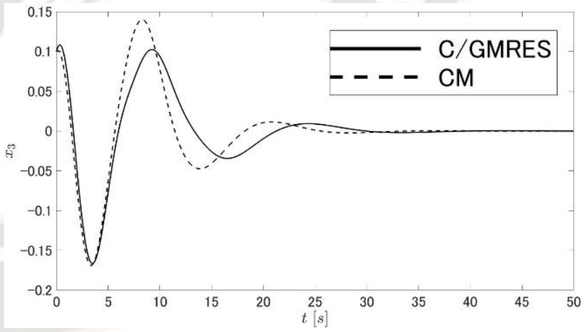

Figure 4: Time responses of x_3.

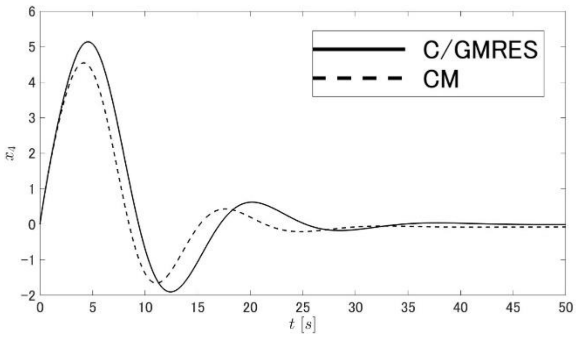

Figure 5: Time responses of x_4.

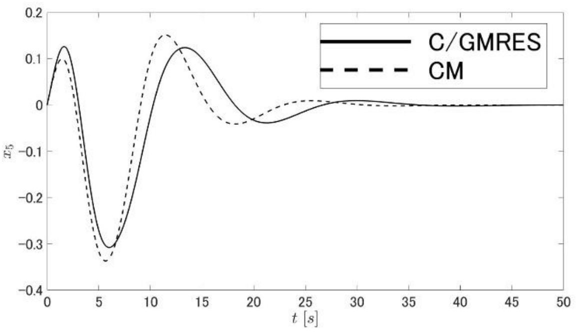

Figure 6: Time responses of x_5.

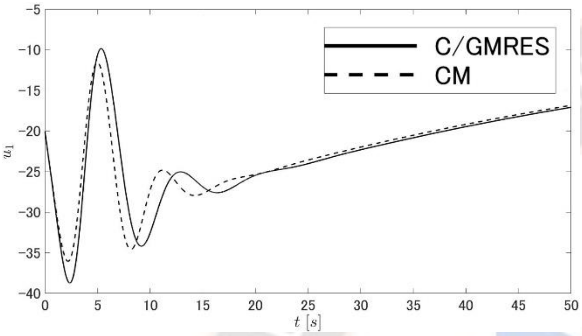

Figure 7: Time responses of u_1.

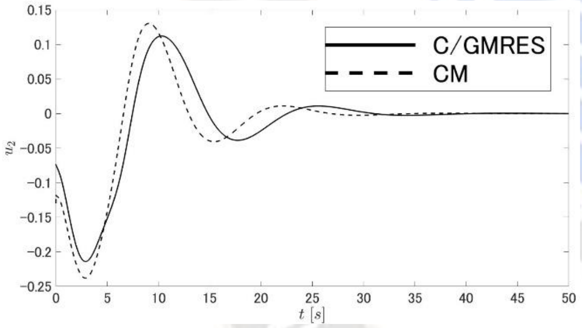

Figure 8: Time responses of u_2.

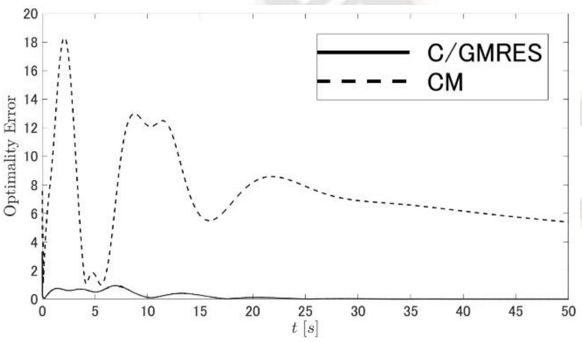

Figure 9: Time responses of optimality error.

CONCLUSION

In this study, an automatic stabilization method is proposed based on nonlinear model predictive control (NMPC) for vehicle dynamics. In this problem, the nonlinearity of vehicle dynamics cannot be neglected. Hence, a nonlinear tire model was employed in this study to consider the realistic behavior of vehicle dynamics. The MPC method previously proposed for control of autonomous vehicle applied the C/GMRES algorithm for numerically solving the nonlinear model predictive control problem. The contraction mapping algorithm is known to be faster than the C/GMRES algorithm. The contraction mapping method is useful for solving the nonlinear model predictive control problem within a short sampling period. Then, the contraction mapping method was applied in this study to solving the nonlinear model predictive control problem of autonomous vehicle. In this study, a nonlinear model predictive control method with a fast numerical solution algorithm for designing an automatic stabilization control system was established. The effectiveness of the proposed method was verified by numerical simulations.

REFERENCES

[1] J. Liu, P. Jayakumar, J.L. Stein and T. Ersal: A nonlinear model predictive control formulation for obstacle avoidance in high-speed autonomous ground vehicles in unstructured environments, Vehicle System Dynamics, Vol. 56, Issue 6, pp. 853-882, 2018.

[2] M. Ataei, A. Khajepour and S. Jeon: Model Predictive Control for integrated lateral stability, traction/braking control, and rollover prevention of electric vehicles, Vehicle System Dynamics, Vol. 58, No. 1, pp. 49-73, 2019.

[3] M.S. Basrah, E. Siampis, E. Velenis, D. Cao and S. Longo: Wheel slip control with torque blending using linear and nonlinear model predictive control, Vehicle System Dynamics, Vol. 55, Issue 11, pp. 1665-1685, 2017.

[4] S.A. Sajadi-Alamdari, H. Voos and M. Darouach: Nonlinear Model Predictive Control for Ecological Driver Assistance Systems in Electric Vehicles, Robotics and Autonomous Systems, Vol. 112, No. 2, pp. 291-303, 2019.

[5] P. Falcone, F. Borrelli, J. Asgari, H.E. Tseng and D. Hrovat, Predictive Active Steering Control for Autonomous Vehicle Systems, IEEE Transaction on Control Systems Technology, Vol. 15, No. 3, pp. 566-580, 2007.

[6] H. L. Zhou and Z. Y. Liu, Design of Vehicle Yaw Stability Controller based on Model Predictive Control, Proceeding of IEEE Intelligent Vehicles Symposiums, pp. 66-71, 2009.

[7] Li. Gang, Z.C. Fu, Z. Hong-Yu and H. Wei, Vehicle Active Front Steering and Yaw Moment Integrated Control, Proceeding of IEEE on Transportation, Mechanical, and Electrical Engineering, pp. 787-790, 2011.

[8] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control for Hot Strip Mill Cooling Systems, IEEE/ASME Transactions on Mechatronics, Vol. 18, No. 3, pp. 998-1005, 2013.

[9] T. Hashimoto, R. Satoh and T. Ohtsuka, Receding Horizon Control for Spatiotemporal Dynamic Systems, Mechanical Engineering Journal, Vol. 3, No. 2, 15-00345, 2016.

[10] T. Hashimoto, Solutions to Probabilistic Constrained Optimal Control Problems Using Concentration Inequalities, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, Vol. 10, No. 10, pp.441-446, 2016.

[11] T. Hashimoto, Receding Horizon Control for a Class of Discrete-time Nonlinear Implicit Systems, Proceedings of IEEE Conference on Decision and Control, pp. 5089-5094, 2014.

[12] T. Hashimoto, Implicit Model Predictive Control for Discretized Brayton-Moser Equations, Proceedings of the 22nd International Conference on Control, Automation, and Systems, TA3-2, 2022.

[13] T. Baba, T. Hashimoto, L.-K. Chen, Model Predictive Control for Stabilization of Vehicle Nonlinear Dynamics to Avoid the Second Collision Accident, Advances in Materials and Processing Technologies, Vol. 6, No. 1, pp.104-114, 2020.

[14] T. Ohtsuka: A Continuation/GMRES Method for Fast Computation of Nonlinear Receding Horizon Control, Automatica, Vol. 40, No. 4, pp. 563-574, 2004.

[15] T. Hashimoto, Y. Yoshioka, T. Ohtsuka, Receding Horizon Control with Numerical Solution for Nonlinear Parabolic Partial Differential Equations, IEEE Transactions on Automatic Control, Vol. 58, No. 3, pp.725-730, 2013.

[16] H. B. Pacejka: A New Tire Model with an Application in Vehicle Dynamics Studies, SAE Technical Paper 890087, 1989.