

PERFECT MATCHING IN COMPLETE INTUITIONISTIC FUZZY GRAPHS

¹ Mahadevaswamy B.S

Associate professor of Mathematics

Maharanis science college for women (Autonomous),
Mysore, Karnataka, India.

Email Id: mahadevaswamybs80@gmail.com

Abstract— A induced sub graph $\langle H \rangle$ of a intuitionistic fuzzy graph $G(V, E)$ is said to be a matching if every vertex of $G(V, E)$ is incident with at most a vertex in $\langle H \rangle$. A induced fuzzy sub graph $\langle H \rangle$ of a intuitionistic fuzzy graph $G(V, E)$ is said to be a perfect matching if every vertex of $G(V, E)$ is incident with exactly a vertex in $\langle H \rangle$. We shall define a perfect matching in intuitionistic fuzzy graphs in this paper. We also look into the matching number's bounds and characteristics in a variety of intuitionistic fuzzy graphs

Keywords- IFG, matching, perfect matching, maximal matching

I. INTRODUCTION

A induced sub graph $\langle H \rangle$ of a IFG $G(V, E)$ is said to be a matching if every vertex of $G(V, E)$ is incident with at most a vertex in $\langle H \rangle$. A maximal matching of a IFG $G(V, E)$ is a matching that cannot be expanded by adding a new edge in $\langle H \rangle$. The matching number of the IFG is minimum among all the maximal matching in $G(V, E)$.

An intuitionistic fuzzy graph (IFG) is of form $G = (V, E)$, where $V = \{v_1, v_2, v_3, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for all $v_i \in V$, for every $v_i \in V, (i = 1, 2, \dots, n)$, $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j)),$$

$$\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j))$$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Let $G = (V, E)$ be an IFG, then the vertex cardinality of V defined by $|V| = \sum_{v \in V} \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2}$ for all $v_i \in V$.

Let $G = (V, E)$ be an IFG, then the edge cardinality of E defined by $|E| = \sum_{v_i, v_j \in E} \frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}$ for all $(v_i, v_j) \in E$.

An IFG, $G = (V, E)$ is said to be complete IFG if $\mu_2 = (\mu_i \wedge \mu_j), \gamma_2 = (\gamma_i \wedge \gamma_j)$ for every $(v_i, v_j) \in V$.

II. PERFECT MATCHING IN INTUITIONISTIC FUZZY GRAPH

This section covers the idea of perfect matching in IFGs as well as some of the characteristics and limitations of a perfect matching number in IFGs..

Definition 2.1. A induced sub graph $\langle H \rangle$ of a IFG $G(V, E)$ is said to be a perfect matching if every vertex of $G(V, E)$ is incident with exactly a vertex in $\langle H \rangle$. A maximal perfect matching of a IFG $G(V, E)$ is a perfect matching that cannot be expanded by adding a new strong edge in $\langle H \rangle$. The perfect matching number of the IFG is minimum among all the maximal perfect matching in $G(V, E)$.

Example 2.1.

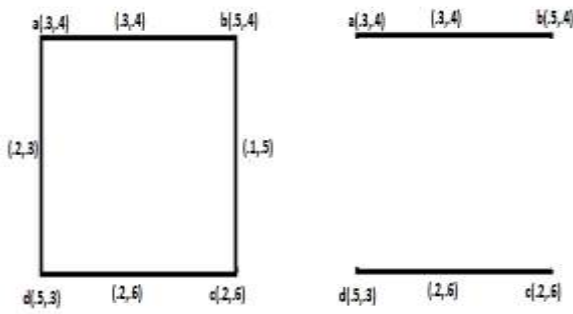


Figure 2.1: Maximal perfect matching of aIFG

In this paper we will define a perfect matching in IFG. Further we investigate the bounds and properties of matching number in various IFG.

Theorem 2.1: For a complete IFG K_n then the

matching number of K_n , $\nu_f(G) = \sum_{i=1}^{\frac{n}{2}} v_i$, where v_i is the vertex having the i^{th} minimum membership value and n is even.

Proof: Let K_n be a complete IFG with even number of vertices. In K_n there is a strong edge between every pair of vertices. Construct the label of the vertex v_i for the vertex having the i^{th} minimum membership value. If the two vertices having the same membership value the label of the vertices are v_i & v_{i+1} . In K_n there is a strong edge between every pair of vertices this implies the edge $\mu(v_i v_j) = \sigma(v_i)$, if $i < j$. Therefore the set of all edges

$$M(G) = \left\{ v_1 v_j, v_2 v_j, \dots, v_{\left(\frac{n}{2}\right)} v_j \mid \left(\frac{n}{2} + 1\right) \leq j \leq n \right\} \text{ cover all}$$

vertices in K_n . Note that there no adjacent edges in this set.

This implies $M(G)$ is a perfect matching of the (CIFG) K_n .

$$M_{IF}(G) = \left\{ v_1 v_j, v_2 v_j, \dots, v_{\left(\frac{n}{2}\right)} v_j \mid \left(\frac{n}{2} + 1\right) \leq j \leq n \right\}$$

$$|M_{IF}(G)| = \left| (v_1 v_j), (v_2 v_j), \dots, (v_{\left(\frac{n}{2}\right)} v_j) \right|$$

$$\nu_{IF}(G) = v_1 + v_2 + v_3 + \dots + v_{\left(\frac{n}{2}\right)}, \text{ Since is a strong edge in } K_n$$

$$\nu_{IF}(G) = \sum_{i=1}^{\frac{n}{2}} v_i.$$

Example 2.2.

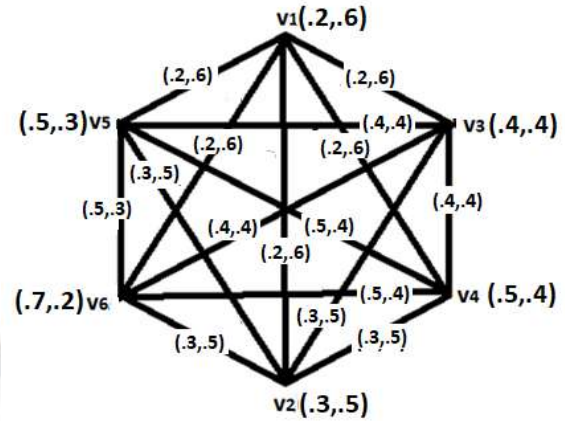


Figure 2.2: Complete IFG K_6

The perfect matching $M_{IF}(G)$ of (CIFG) K_6 is $M_{IF}(G) = \{v_1 v_4, v_2 v_5, v_3 v_6\}$ and the matching number $\nu_{IF}(G) = 1.2$.

Definition 2.2: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two IFGs with $V_1 \cap V_2 = \emptyset$ and $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$. Then the union of IFGs G_1 and G_2 is an IFG defined by

$$(\mu_{11} \cup \mu_{21})(v) = \begin{cases} \mu_{11}(v) & \text{if } v \in V_1 \\ \mu_{21}(v) & \text{if } v \in V_2 \end{cases}$$

$$(\gamma_{11} \cup \gamma_{21})(v) = \begin{cases} \gamma_{11}(v) & \text{if } v \in V_1 \\ \gamma_{21}(v) & \text{if } v \in V_2 \end{cases} \quad \text{and}$$

$$(\mu_{12} \cup \mu_{22})(v_i v_j) = \begin{cases} \mu_{12}(v_i v_j) & \text{if } v_i v_j \in E_1 \\ \mu_{22}(v_i v_j) & \text{if } v_i v_j \in E_2 \end{cases}$$

$$(\gamma_{12} \cup \gamma_{22})(v_i v_j) = \begin{cases} \gamma_{12}(v_i v_j) & \text{if } v_i v_j \in E_1 \\ \gamma_{22}(v_i v_j) & \text{if } v_i v_j \in E_2 \end{cases}$$

where (μ_{11}, γ_{21}) and (μ_{11}, γ_{21}) refer the vertex membership and non-membership of G_1 and G_2 respectively; (μ_{12}, γ_{22}) and (μ_{22}, γ_{22}) refer the edge membership and non-membership of G_1 and G_2 respectively.

Theorem 2.2: For a union $G_1 \cup G_2$ of two IFG $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ having same and even crisp order n , then the matching number of $G_1 \cup G_2$ is $\nu_{IF}(G_1 + G_2) = \nu_{IF}(G_1) + \nu_{IF}(G_2)$.

Proof: Let $G_1 \cup G_2$ be a union of two IFG $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ having same even crisp order n . Let $M_{IF}(G_1)$ & $M_{IF}(G_2)$ be a minimal perfect matching of $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ respectively. Therefore every vertex in $G_1(V_1, E_1)$ is incident with exactly a vertex in

$\langle M_{IF}(G_1) \rangle$ and every vertex in $G_2(V_2, E_2)$ is incident with exactly a vertex in $\langle M_{IF}(G_2) \rangle$. This implies every vertex in $G_1 \cup G_2$ is incident with exactly a vertex in $\langle M_{IF}(G_1) \cup M_{IF}(G_2) \rangle$. Hence $M_{IF}(G_1) \cup M_{IF}(G_2)$ is a minimal perfect matching of $G_1 \cup G_2$. This implies

$$\begin{aligned} \nu_{IF}(G_1 + G_2) &= |M_{IF}(G_1) \cup M_{IF}(G_2)| \\ \nu_{IF}(G_1 + G_2) &= \nu_{IF}(G_1) + \nu_{IF}(G_2) \end{aligned}$$

Example 2.3:

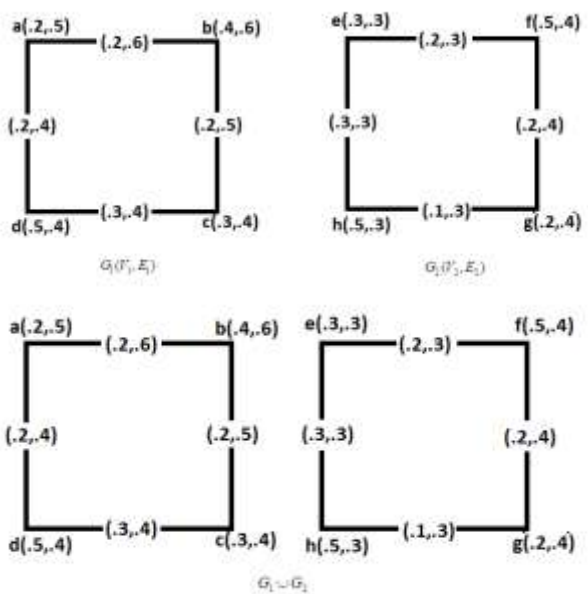


Figure 2.4: Union of two IFG $G_1 \cup G_2$

The $M_{IF}(G)$ is a perfect matching in Join of two IFG $G_1 \cup G_2$ is $M_{IF}(G) = \{ab, cd, eh, fg\}$ and the IF matching number $\nu_{IF}(G_1 \cup G_2) = 1.3$.

$$M_{IF}(G) = \left\{ (u_1 v_n), (u_2 v_{(n-1)}), \dots, (u_{(n/2)} v_{(n/2+1)}), (u_{(n/2+1)} v_{(n/2)}), \dots, (u_{(n-1)} v_2), (u_n v_1) \right\}$$

cover all vertices in $G_1(V_1, E_1) \& G_2(V_2, E_2)$.

Note that there $M_{IF}(G)$ is a maximal matching of $G_1 + G_2$.

This implies $M_{IF}(G)$ is a perfect matching of $G_1 + G_2$.

$$M_{IF}(G) = \left\{ (u_1 v_n), (u_2 v_{(n-1)}), \dots, (u_{(n/2)} v_{(n/2+1)}), (u_{(n/2+1)} v_{(n/2)}), \dots, (u_{(n-1)} v_2), (u_n v_1) \right\}$$

$$|M_{IF}(G)| = |(u_1 v_n)| + |(u_2 v_{(n-1)})| + \dots + |(u_{(n/2)} v_{(n/2+1)})| + |(u_{(n/2+1)} v_{(n/2)})| + \dots + |(u_{(n-1)} v_2)| + |(u_n v_1)|$$

$$|M_{IF}(G)| = |u_1| + |u_2| + \dots + |u_{(n/2)}| + |v_{(n/2)}| + \dots + |v_2| + |v_1|$$

$$\nu_{IF}(G) = \sum_{i=1}^{(n/2)} u_i + \sum_{i=1}^{(n/2)} v_i$$

Example 2.4:

Definition 2.3: The join of two IFGs G_1

and G_2 is an IFG $G = G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2 \cup E)$ defined by

$$\begin{aligned} (\mu_{11} + \mu_{21})(v) &= (\mu_{11} \cup \mu_{21})(v) \quad \text{if } v \in V_1 \cup V_2 \\ (\gamma_{11} + \gamma_{21})(v) &= (\gamma_{11} \cup \gamma_{21})(v) \quad \text{if } v \in V_1 \cup V_2 \\ (\mu_{12} + \mu_{22})(v_i v_j) &= (\mu_{12} \cup \mu_{22})(v_i v_j) \quad \text{if } v_i v_j \in E_1 \cup E_2 \\ &= \mu_{11}(v_i) \wedge \mu_{21}(v_j) \quad \text{if } v_i \in V_1 \text{ and } v_j \in V_2 \\ &\text{and} \\ (\gamma_{12} + \gamma_{22})(v_i v_j) &= (\gamma_{12} \cup \gamma_{22})(v_i v_j) \quad \text{if } v_i v_j \in E_1 \cup E_2 \\ &= \gamma_{11}(v_i) \wedge \gamma_{21}(v_j) \quad \text{if } v_i \in V_1 \text{ and } v_j \in V_2 \end{aligned}$$

Theorem 2.3: For a join $G_1 + G_2$ of two IFG $G_1(V_1, E_1) \& G_2(V_2, E_2)$ having same and even crisp order n , then the matching number of $G_1 + G_2$ is

$$\nu_{IF}(G) = \sum_{i=1}^{n/2} u_i + \sum_{i=1}^{n/2} v_i.$$

Proof: Let $G_1 + G_2$ be a join of two IFG $G_1(V_1, E_1) \& G_2(V_2, E_2)$ having same even crisp order n . Construct the label of the vertex $u_i \& v_i$ for the vertex having the i^{th} minimum membership value in $G_1(V_1, E_1) \& G_2(V_2, E_2)$ respectively. If the two vertices having the same membership value the labels of the vertices are $v_i \& v_{i+1}$. In $G_1 + G_2$ there is a strong edge between vertices in $G_1(V_1, E_1) \& G_2(V_2, E_2)$. Therefore the set of all edges

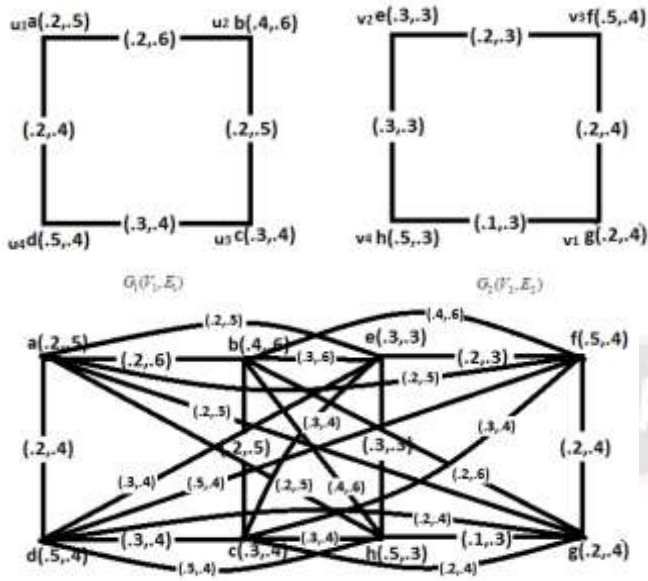


Figure 2.4: Join of two IFG $G_1 + G_2$

The perfect matching $M_{IF}(G)$ of Join of two fuzzy graph $G_1 + G_2$ is $M_{IF}(G) = \{ah, bf, ce, dg\}$ and the fuzzy matching number $\nu_{IF}(G_1 + G_2) = 1.6$.

Note: Product of edges and vertex in IFG is defined by $(ab) \times c = ((ac)(bc))$, where $(ab) \in E$ & $c \in V$

Definition 2.4. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is two IFG on V_1 and V_2 respectively. Then the strong product of G_1 and G_2 , denoted by $(\sigma_1 * \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$, is the IFG on $V_1 \times V_2$ defined as follows

$$(G_1 * G_2) = (V, E)$$

$$(\mu_{11} * \mu_{21})(u_1, u_2) = \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$

Where

$$(\gamma_{11} * \gamma_{21})(u_1, u_2) = \gamma_{11}(u_1) \vee \gamma_{21}(u_2)$$

and

$$(\mu_{12} * \mu_{22})((u_1, u_2)(v_1, v_2)) = \mu_{12}(u_1v_1) \wedge \mu_{22}(u_2v_2), \text{ if } (u_1v_1) \in E_1$$

$$(\gamma_{12} * \gamma_{22})((u_1, u_2)(v_1, v_2)) = \gamma_{12}(u_1v_1) \vee \gamma_{22}(u_2v_2), \text{ if } (u_1v_1) \in E_1$$

Theorem 2.4: For a strong product $G_1 * G_2$ of two IFG $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ having even crisp order m & n respectively, then the matching number of $G_1 \times G_2$ is $\nu_{IF}(G_1 * G_2) = |M_{IF}(G_1) \times M_{IF}(G_2)V_2|$, where $M_{IF}(G_1)$ & $M_{IF}(G_2)$ are perfect matching of $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively.

Proof: Let $G_1 * G_2$ be a strong product of two IFG $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ having even crisp order m & n

respectively. Let $M_{IF}(G_1)$ & $M_{IF}(G_2)$ are maximal perfect matching of $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively. Now we prove $(M_{IF}(G_1) \times V_2)$ be a maximal perfect matching of $G_1 \circ G_2$. Note that every edges in $M_{IF}(G_1)$ are strong edges in $G_1 \circ G_2$ and $M_{IF}(G_1)$ cover all the vertices in $G_1 * G_2$.

Let $(u_1v_1) \in M_{IF}(G_1)$ & $(u_2v_2) \in M_{IF}(G_2)$ this implies $(u_1u_2)(v_1v_2) \in G_1 * G_2$ is a strong edge in $G_1 * G_2$, since (u_1v_1) & (u_2v_2) is a strong edge in $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively. The set of all strong edges of the form $(u_1u_2)(v_1v_2)$ covers all the vertices in $G_1 * G_2$. Therefore $(M_{IF}(G_1) \times M_{IF}(G_2))$ is a perfect matching of $G_1 * G_2$. Note that every vertex of $G_1 * G_2$ is incident with exactly a vertex in $(M(G_1) \times M(G_2))$. Since $M_{IF}(G_1)$ & $M_{IF}(G_2)$ is a maximal perfect matching of $G_2(\sigma_2, \mu_2)$. Hence $(M_{IF}(G_1) \times M_{IF}(G_2))$ is a maximal perfect matching of $G_1 * G_2$.

$$M_{IF}(G_1 * G_2) = (M_{IF}(G_1) \times M_{IF}(G_2))$$

$$|M_{IF}(G_1 * G_2)| = |(M_{IF}(G_1) \times M_{IF}(G_2))|$$

$$\nu_{IF}(G_1 * G_2) = |(M_{IF}(G_1) \times M_{IF}(G_2))|$$

Example 2.5:

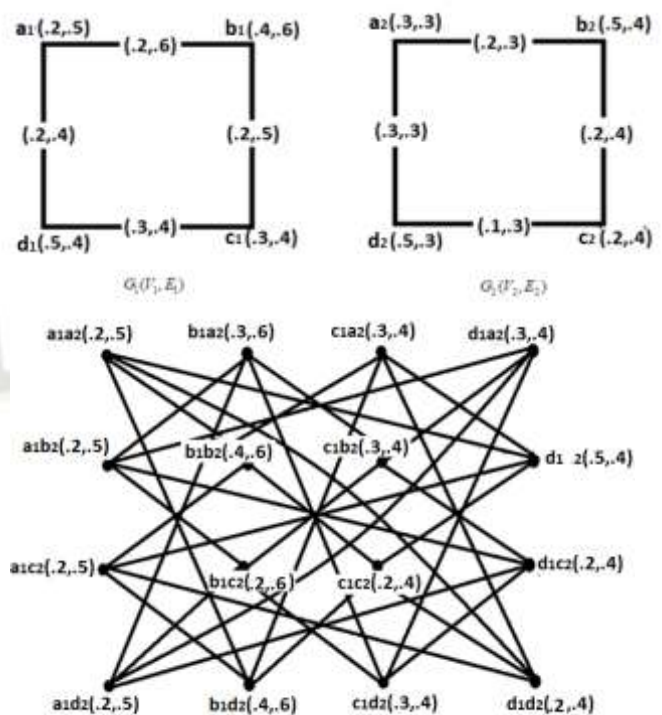


Figure 2.4: Strong product of two IFG $G_1 * G_2$

Edge	$\mu_{12} * \mu$	Edge	$\mu_{12} * \mu$	Edge	$\mu_{12} * \mu$		4)		5)		4)
$(a_1a_2)(b_1d_1)$	(0.2,0.6)	$(a_1c_2)(d_1d_2)$	(0.2,0.4)	$(b_1d_2)(c_1d_1)$	(0.1,0.5)	$(a_1b_2)(d_1d_2)$	(0.2,0.4)	$(b_1b_2)(c_1d_1)$	(0.2,0.5)	$(c_1c_2)(d_1d_2)$	(0.1,0.4)
$(a_1a_2)(d_1d_2)$	(0.2,0.4)	$(a_1d_2)(d_1d_2)$	(0.2,0.4)	$(b_1d_2)(c_1d_1)$	(0.2,0.5)	$(a_1c_2)(b_1d_1)$	(0.1,0.6)	$(b_1b_2)(c_1d_1)$	(0.2,0.5)	$(c_1d_2)(d_1d_2)$	(0.3,0.4)
$(a_1a_2)(d_1d_2)$	(0.2,0.4)	$(a_1d_2)(b_1d_1)$	(0.2,0.6)	$(c_1a_2)(d_1d_2)$	(0.3,0.4)	$(a_1c_2)(d_1d_2)$	(0.1,0.4)	$(b_1c_2)(c_1d_1)$	(0.2,0.5)	$(c_1d_2)(d_1d_2)$	(0.3,0.4)
$(a_1a_2)(b_1d_1)$	(0.2,0.6)	$(a_1d_2)(b_1d_1)$	(0.1,0.6)	$(c_1a_2)(d_1d_2)$	(0.2,0.4)	$(a_1c_2)(b_1d_1)$	(0.2,0.6)	$(b_1c_2)(c_1d_1)$	(0.1,0.5)		
$(a_1b_2)(b_1d_1)$	(0.2,0.6)	$(a_1d_2)(d_1d_2)$	(0.1,0.4)	$(c_1b_2)(d_1d_2)$	(0.2,0.4)						
$(a_1b_2)(b_1d_1)$	(0.2,0.6)	$(b_1a_2)(c_1d_1)$	(0.2,0.5)	$(c_1b_2)(d_1d_2)$	(0.2,0.4)						
$(a_1b_2)(d_1d_2)$	(0.2,0.4)	$(b_1a_2)(c_1d_1)$	(0.2,0.4)	$(c_1c_2)(d_1d_2)$	(0.2,0.4)						

The perfect matching $M_{IF}(G_1) \& M_{IF}(G_2)$ of twoIFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ is $M_{IF}(G_1) = \{a_1b_1, c_1d_1\} \& M_{IF}(G_2) = \{a_2d_2, b_2c_2\}$ and The perfect matching $M_{IF}(G)$ of strong product of twoIFG $G_1 * G_2$ is

$$M_{IF}(G) = \{((a_1a_2)(b_1d_2)), ((b_1a_2)(a_1d_2)), ((a_1b_2)(b_1c_2)), ((b_1b_2)(a_1c_2)), ((c_1a_2)(d_1d_2)), ((d_1a_2)(c_1d_2)), ((c_1b_2)(d_1c_2)), ((d_1b_2)(c_1c_2))\}$$

IFG on $V_1 \times V_2$ defined as follows

and the IF matching number $\nu_{IF}(G_1 * G_2) = 2.85$.

Definition 2.5. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ is two IFG on V_1 and V_2 respectively. Then the Cartesian product of G_1 and G_2 , denoted by $G_1 \times G_2$, is the

$$G_1 \times G_2 = (V_1 \times V_2, E_1 \times E_2) \text{ where}$$

$$(\mu_{11} \times \mu_{21})(u_1, u_2) = \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$

$$(\gamma_{11} \times \gamma_{21})(u_1, u_2) = \gamma_{11}(u_1) \vee \gamma_{21}(u_2) \text{ and}$$

$$(\mu_{12} \times \mu_{22})((u_1, u_2), (v_1, v_2)) = \begin{cases} \mu_{11}(u_1) \wedge \mu_{22}(u_2v_2) & \text{if } u_1 = v_1 \\ \mu_{21}(u_2) \wedge \mu_{12}(u_1v_1) & \text{if } u_2 = v_2 \\ 0 & \text{otherwise} \end{cases}$$

$$(\gamma_{12} \times \gamma_{22})((u_1, u_2), (v_1, v_2)) = \begin{cases} \gamma_{11}(u_1) \vee \gamma_{22}(u_2v_2) & \text{if } u_1 = v_1 \\ \gamma_{21}(u_2) \vee \gamma_{12}(u_1v_1) & \text{if } u_2 = v_2 \\ 0 & \text{otherwise} \end{cases}$$

Theorem 2.5: For a Cartesian product $G_1 \times G_2$ of twoIFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ having even crisp order $m \& n$ respectively, then the matching number of $G_1 \times G_2$ is $\nu_{IF}(G_1 \times G_2) = |M_{IF}(G_1) \times V_2| \wedge |V_1 \times M_{IF}(G_2)|$, where $M(G_1) \& M(G_2)$ are perfect matching of $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ respectively.

Proof: Let $G_1 \times G_2$ be a Cartesian product of twoIFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ having even crisp order $m \& n$ respectively. Let $M_{IF}(G_1) \& M_{IF}(G_2)$

are maximal perfect matching of $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ respectively. Now we prove $(M(G_1) \times V_2)$ be a maximal perfect matching of $G_1 \times G_2$. Note that every edges in $M_{IF}(G_1)$ are strong edges in $G_1 \times G_2$ and $M_{IF}(G_1)$ cover all the vertices in $G_1(\sigma_1, \mu_1)$. Let $(u_1v_1) \in M(G_1)$ and $u_2 \in V_2$ this implies $(u_1u_2)(v_1u_2) \in G_1 \times G_2$ is a strong edge in $G_1 \times G_2$, since (u_1v_1) is a strong edge in $G_1(\sigma_1, \mu_1)$. The set of all strong edges of the form $(u_1v_1) \times u_2$ covers all the vertices in $G_1 \times G_2$. Therefore $(M_{IF}(G_1) \times V_2)$ is a matching of $G_1 \times G_2$. Note that every vertex of $G_1 \times G_2$ is

incident with exactly a vertex in $\langle M_{IF}(G_1) \times V_2 \rangle$. Since $M_{IF}(G_1)$ is a maximal perfect matching of $G_1(\sigma_1, \mu_1)$. Hence $(M_{IF}(G_1) \times V_2)$ is a maximal perfect matching of $G_1 \times G_2$.

Let $(u_2 v_2) \in M_{IF}(G_2)$ and $u_1 \in V_1$ this implies $(u_1 u_2)(u_1 v_2) \in G_1 \times G_2$ is a strong edge in $G_1 \times G_2$, since $(u_2 v_2)$ is a strong edge in $G_2(\sigma_2, \mu_2)$. The set of all strong edges of the form $u_1 \times (u_2 v_2)$ covers all the vertices in $G_1 \times G_2$. Therefore $(V_1 \times M_{IF}(G_2))$ is a perfect matching of $G_1 \times G_2$. Note that every vertex of $G_1 \times G_2$ is incident with exactly a vertex in $\langle V_1 \times M_{IF}(G_2) \rangle$. Since $M_{IF}(G_2)$ is a maximal perfect matching of $G_2(\sigma_2, \mu_2)$. Hence $(V_1 \times M_{IF}(G_2))$ is a maximal perfect matching of $G_1 \times G_2$.

$$M_{IF}(G_1 \times G_2) = (M_{IF}(G_1) \times V_2) \cup (V_1 \times M_{IF}(G_2))$$

$$|M_{IF}(G_1 \times G_2)| = |(M_{IF}(G_1) \times V_2)| + |(V_1 \times M_{IF}(G_2))|$$

$$v_{IF}(G_1 \times G_2) = |(M_{IF}(G_1) \times V_2)| + |(V_1 \times M_{IF}(G_2))|$$

Example 2.4:

The perfect matching $M(G_1) \& M(G_2)$ of two IFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ is $M_f(G_1) = \{a_1 b_1, c_1 d_1\} \& M_f(G_2) = \{a_2 d_2, b_2 c_2\}$ and The perfect matching $M_f(G)$ of Cartesian product of two IFG $G_1 \times G_2$ is

$$M_f(G) = \{((a_1 a_2)(b_1 a_2)), ((c_1 a_2)(d_1 a_2)), ((a_1 b_2)(b_1 b_2)), ((c_1 b_2)(d_1 b_2)), ((a_1 c_2)(b_1 c_2)), ((c_1 c_2)(b_1 d_2)), ((c_1 d_2)(d_1 d_2))\}$$

and the IF matching number $v_{IF}(G_1 \times G_2) = 2.85$.

Definition 2.6. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two IFG on V_1 and V_2 respectively. Then the composition of G_1 and G_2 , denoted by $(\sigma_1 \circ \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$, is the IFG on

$$(\mu_{12} \times \mu_{22})((u_1, u_2), (v_1, v_2)) = \begin{cases} \mu_{11}(u_1) \wedge \mu_{22}(u_2 v_2) & \text{if } u_1 = v_1 \\ \mu_{21}(u_2) \wedge \mu_{12}(u_1 v_1) & \text{if } u_2 = v_2 \\ \mu_{12}(u_1 v_1) \wedge \mu_{22}(u_2 v_2), & \text{if } (u_1 v_1) \in E_1 \& (u_2 v_2) \in E_2 \end{cases}$$

$$(\gamma_{12} \times \gamma_{22})((u_1, u_2), (v_1, v_2)) = \begin{cases} \gamma_{11}(u_1) \vee \gamma_{22}(u_2 v_2) & \text{if } u_1 = v_1 \\ \gamma_{21}(u_2) \vee \gamma_{12}(u_1 v_1) & \text{if } u_2 = v_2 \\ \gamma_{12}(u_1 v_1) \vee \gamma_{22}(u_2 v_2), & \text{if } (u_1 v_1) \in E_1 \& (u_2 v_2) \in E_2 \end{cases}$$

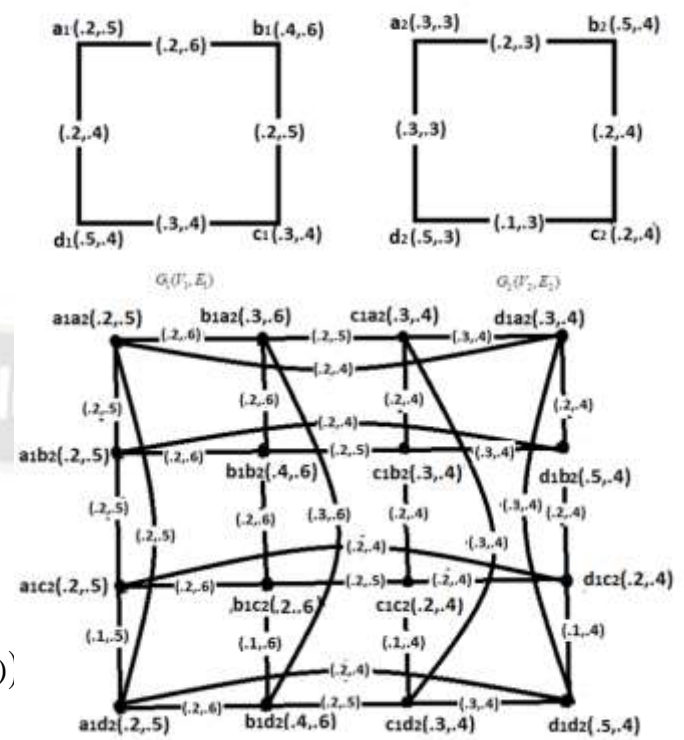


Figure 2.5: Cartesian product of two IFG $G_1 \times G_2$

The perfect matching $M(G_1) \& M(G_2)$ of two IFG

$G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ is

$M_f(G_1) = \{a_1 b_1, c_1 d_1\} \& M_f(G_2) = \{a_2 d_2, b_2 c_2\}$ and

The perfect matching $M_f(G)$ of Cartesian product of two IFG

$G_1 \times G_2$ is

$$M_f(G) = \{((a_1 a_2)(b_1 a_2)), ((c_1 a_2)(d_1 a_2)), ((a_1 b_2)(b_1 b_2)), ((c_1 b_2)(d_1 b_2)), ((a_1 c_2)(b_1 c_2)), ((c_1 c_2)(b_1 d_2)), ((c_1 d_2)(d_1 d_2))\}$$

$V_1 \times V_2$ defined as follows

$$G_1 \circ G_2 = (V_1 \circ V_2, E_1 \circ E_2)$$

$$(\mu_{11} \times \mu_{21})(u_1, u_2) = \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$

$$(\gamma_{11} \times \gamma_{21})(u_1, u_2) = \gamma_{11}(u_1) \vee \gamma_{21}(u_2)$$

Where

and

Theorem 2.6: For a composition product $G_1 \circ G_2$ of

two IFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ having even crisp order

$m \& n$ respectively, then the matching number of $G_1 \circ G_2$ is

$$v_{IF}(G_1 \circ G_2) = |M_{IF}(G_1) \times V_2| + |V_1 \times M_{IF}(G_2)| + |M_{IF}(G_1) \times M_{IF}(G_2)|$$

, where $M_{IF}(G_1) & M_{IF}(G_2)$ are perfect matching of $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively.

Proof: Let $G_1 \circ G_2$ be a composition of two IFG $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ having even crisp order m & n respectively. Let $M_{IF}(G_1) & M_{IF}(G_2)$ are maximal perfect matching of $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively. Now we prove $(M_{IF}(G_1) \times V_2)$ be a maximal perfect matching of $G_1 \circ G_2$. Note that every edges in $M_{IF}(G_1)$ are strong edges in $G_1 \circ G_2$ and $M_{IF}(G_1)$ cover all the vertices in $G_1(\sigma_1, \mu_1)$. Let $(u_1 v_1) \in M_{IF}(G_1)$ and $u_2 \in V_2$ this implies $(u_1 u_2)(v_1 u_2) \in G_1 \circ G_2$ is a strong edge in $G_1 \circ G_2$, since $(u_1 v_1)$ is a strong edge in $G_1(\sigma_1, \mu_1)$. The set of all strong edges of the form $(u_1 v_1) \times u_2$ covers all the vertices in $G_1 \circ G_2$. Therefore $(M_{IF}(G_1) \times V_2)$ is a matching of $G_1 \circ G_2$. Note that every vertex of $G_1 \circ G_2$ is incident with exactly a vertex in $\langle M_{IF}(G_1) \times V_2 \rangle$. Since $M_{IF}(G_1)$ is a maximal perfect matching of $G_1(\sigma_1, \mu_1)$. Hence $(M_{IF}(G_1) \times V_2)$ is a maximal perfect matching of $G_1 \circ G_2$.

Let $(u_2 v_2) \in M_{IF}(G_2)$ and $u_1 \in V_1$ this implies $(u_1 u_2)(u_1 v_2) \in G_1 \times G_2$ is a strong edge in $G_1 \circ G_2$, since $(u_2 v_2)$ is a strong edge in $G_2(\sigma_2, \mu_2)$. The set of all strong edges of the form $u_1 \times (u_2 v_2)$ covers all the vertices in $G_1 \times G_2$. Therefore $(V_1 \times M_{IF}(G_2))$ is a perfect matching of $G_1 \circ G_2$. Note that every vertex of $G_1 \times G_2$ is incident with exactly a vertex in $\langle V_1 \times M_{IF}(G_2) \rangle$. Since $M_{IF}(G_2)$ is a maximal perfect matching of $G_2(\sigma_2, \mu_2)$. Hence $(V_1 \times M_{IF}(G_2))$ is a maximal perfect matching of $G_1 \circ G_2$.

Let $(u_1 v_1) \in M_{IF}(G_1)$ & $(u_2 v_2) \in M_{IF}(G_2)$ this implies $(u_1 u_2)(v_1 v_2) \in G_1 \times G_2$ is a strong edge in $G_1 \circ G_2$, since $(u_1 v_1) & (u_2 v_2)$ is a strong edge in $G_1(\sigma_1, \mu_1)$ & $G_2(\sigma_2, \mu_2)$ respectively. The set of all strong edges of the form $(u_1 u_2)(v_1 v_2)$ covers all the vertices in $G_1 \circ G_2$. Therefore $(M_{IF}(G_1) \times M_{IF}(G_2))$ is a perfect matching of $G_1 \circ G_2$. Note that every vertex of $G_1 \circ G_2$ is incident with exactly a vertex in $(M_{IF}(G_1) \times M_{IF}(G_2))$. Since $M_{IF}(G_1) & M_{IF}(G_2)$ is a maximal perfect matching of $G_2(\sigma_2, \mu_2)$. Hence $(M_{IF}(G_1) \times M_{IF}(G_2))$ is a maximal perfect matching of $G_1 \circ G_2$.

$$M_{IF}(G_1 \times G_2) = (M_{IF}(G_1) \times V_2) \text{ or } (V_1 \times M_{IF}(G_2)) \text{ or } (M_{IF}(G_1) \times M_{IF}(G_2))$$

$$|M_{IF}(G_1 \times G_2)| = |(M_{IF}(G_1) \times V_2)| \wedge |(V_1 \times M_{IF}(G_2))| \wedge |(M_{IF}(G_1) \times M_{IF}(G_2))|$$

$$v_{IF}(G_1 \times G_2) = |(M_{IF}(G_1) \times V_2)| \wedge |(V_1 \times M_{IF}(G_2))| \wedge |(M_{IF}(G_1) \times M_{IF}(G_2))|$$

Example 2.5:

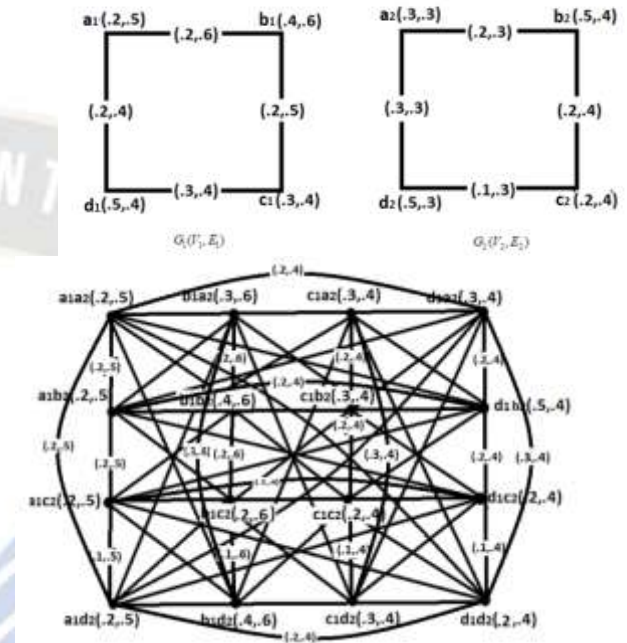


Figure 2.6: Composition of two IFG $G_1 \circ G_2$

Edge	$\mu_{12} \circ \mu_1$	Edge	$\mu_{12} \circ \mu_1$	Edge	$\mu_{12} \circ \mu_1$
$(a_1 a_2)(b_1 b_2)$	(0.2,0.6)	$(a_1 c_2)(d_1 d_2)$	(0.2,0.4)	$(b_1 d_2)(c_1 c_2)$	(0.1,0.5)
$(a_1 a_2)(d_1 d_2)$	(0.2,0.4)	$(a_1 d_2)(d_1 d_2)$	(0.2,0.4)	$(b_1 d_2)(c_1 c_2)$	(0.2,0.5)
$(a_1 a_2)(d_1 d_2)$	(0.2,0.4)	$(a_1 d_2)(b_1 b_2)$	(0.2,0.6)	$(c_1 a_2)(d_1 d_2)$	(0.3,0.4)
$(a_1 a_2)(b_1 b_2)$	(0.2,0.6)	$(a_1 d_2)(b_1 b_2)$	(0.1,0.6)	$(c_1 a_2)(d_1 d_2)$	(0.2,0.4)
$(a_1 b_2)(b_1 b_2)$	(0.2,0.6)	$(a_1 d_2)(d_1 d_2)$	(0.1,0.4)	$(c_1 b_2)(d_1 d_2)$	(0.2,0.4)
$(a_1 b_2)(b_1 b_2)$	(0.2,0.6)	$(b_1 a_2)(c_1 c_2)$	(0.2,0.5)	$(c_1 b_2)(d_1 d_2)$	(0.2,0.4)
$(a_1 b_2)(d_1 d_2)$	(0.2,0.4)	$(b_1 a_2)(c_1 c_2)$	(0.2,0.5)	$(c_1 c_2)(d_1 d_2)$	(0.2,0.4)
$(a_1 b_2)(d_1 d_2)$	(0.2,0.4)	$(b_1 b_2)(c_1 c_2)$	(0.2,0.5)	$(c_1 c_2)(d_1 d_2)$	(0.1,0.4)
$(a_1 c_2)(b_1 b_2)$	(0.1,0.6)	$(b_1 b_2)(c_1 c_2)$	(0.2,0.5)	$(c_1 d_2)(d_1 d_2)$	(0.3,0.4)
$(a_1 c_2)(d_1 d_2)$	(0.1,0.4)	$(b_1 c_2)(c_1 c_2)$	(0.2,0.5)	$(c_1 d_2)(d_1 d_2)$	(0.3,0.4)
$(a_1 c_2)(b_1 b_2)$	(0.2,0.6)	$(b_1 c_2)(c_1 c_2)$	(0.1,0.5)		

The perfect matching $M(G_1) \& M(G_2)$ of two IFG $G_1(\sigma_1, \mu_1) \& G_2(\sigma_2, \mu_2)$ is

$$M_f(G_1) = \{a_1b_1, c_1d_1\} \& M_f(G_2) = \{a_2d_2, b_2c_2\} \text{ and}$$

$$M_f(G) = \{((a_1a_2)(b_1a_2)), ((c_1a_2)(d_1a_2)), ((a_1b_2)(b_1b_2)), ((c_1b_2)(d_1d_2)), ((a_1c_2)(b_1c_2)), ((c_1c_2)(b_1d_2)), ((c_1d_2)(d_1d_2))\} \text{ and the IF}$$

matching number $\nu_{IF}(G_1 \times G_2) = 2.85$.

The perfect matching $M_f(G)$ of Cartesian product of two IFG $G_1 \times G_2$ is

3. Conclusion

We shall define a perfect matching in IFG in this paper. We also look into the characteristics and bounds of matching numbers in different IFG. We will discuss different matching in IFG and examine the characteristics and limitations of the IF matching number in the future.

References:

- [1]. Akram, M & Davvaz, B 2012, 'Strong intuitionistic fuzzy graphs', Filomat, vol.26, pp.177- 196.
- [2]. Akram, M 2011, 'Bipolar fuzzy graphs', Information Sciences, vol.181, pp.5548-5564.
- [3]. Atanasson, K, Intuitionistic Fuzzy Sets: Theory and Applications, Physica-verlag, New York (1999).
- [4]. Bhattacharya P., "Some remarks on Fuzzy Grapjs", Pattern Recognition Lett. 6(1987) 297-302.
- [5]. Cockayne, E. J., R. Dawes, and S. T. Hedetniemi. Total domination in graphs. Networks 10(3)(1980), 211–219.
- [6]. Haynes, T., Hedetniemi, S.T., and Slater, P., Fundamentals of domination in graphs. CRC Press, 1998
- [7]. NagoorGani A and Radha K "Some sequences in Fuzzy Graphs" Far East Journal of Applied Mathematics 31(3) 321-335.
- [8]. Rosenfeld A, Zadeh L.A., Fu and Shimura M., "Fuzzy sets and their applications (1975) 77-95.
- [9]. Rosenfeld. A. Fuzzy graphs. In Fuzzy Sets and their Applications to Cognitive and Decision Processes. Zadeh LA, Fu KS, Shimura M, editors. New York: Academic press; 1975. p. 77–95.
- [10]. Zadeh L.A. Fuzzy sets. Information and Control. 1965; 8:338–53.
- [11]. O Ravi, A senthil kumar R & Hamari CHOUDHI" Decompositions of $\tilde{I}g$ -Continuity via Idealization" Journal of New Results in Science 2014 3 (7), 72-80.
- [12]. S.Tharmar & R.Senthil kumar" Soft Locally Closed Sets in Soft Ideal Topological Spaces" , Transylvanian Review: Vol , No. 10, 10 (XXIV), 1593-1600.
- [13]. SV B.K.K. Priyatharsini, R.SENTHIL KUMAR New footprints of bondage number of connected unicyclic and line graphs, Asia Life Sciences 2017 26 (2), 321-326.
- [14]. K. Prabhavathi, R. Senthilkumar, I. Athal, M. Karthivel" m - $\tilde{I}g$ -CLOSED SETS AND M - $\tilde{I}g$ -CONTINUITY" Jour of Adv Research in Dynamical & Control Systems 10 (4), 112-118, 2018.
- [15]. K. Prabhavathi, R. Senthilkumar, I. Athal, M. Karthivel A Note on $\tilde{I}g$ * g Closed Sets Jour of Adv Research in Dynamical & Control Systems 11 (04-Special Issue 2019